

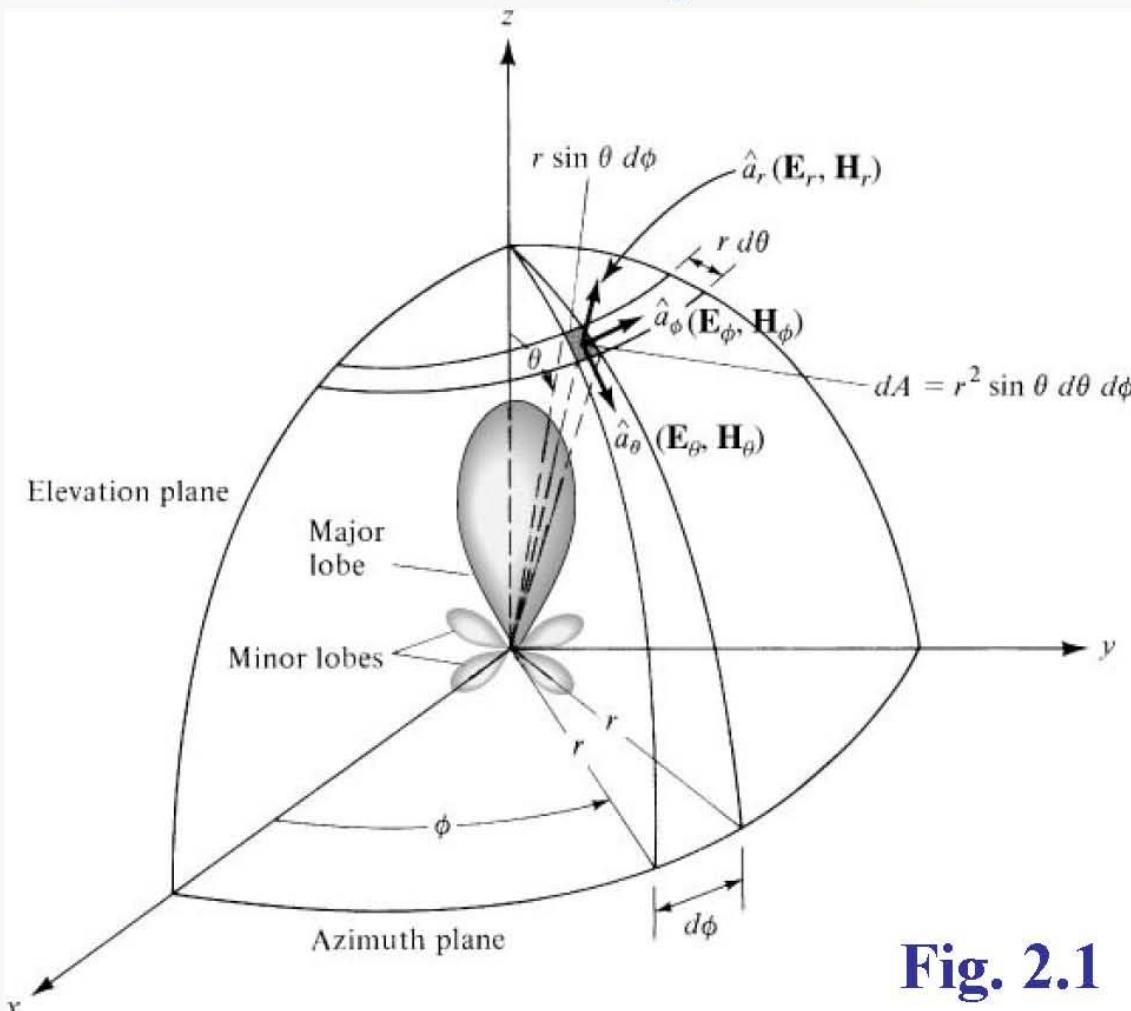
# Chapter 2

## Fundamental Parameters of Antennas

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**Chapter 2**  
*Fundamental Parameters of Antennas*

# Coordinate System



**Fig. 2.1**

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$$\int_0^{2\pi} \left[ \int_0^{\pi} r^2 \sin \theta \, d\theta \right] d\phi = 4\pi r^2$$

$$0 \leq \theta \leq \pi$$

$$0 \leq \phi \leq 2\pi$$



# Radiation Pattern

A mathematical and/or graphical representation of the radiation properties of an antenna, such as the:

- amplitude
- phase
- polarization, etc.

as a function of the angular space coordinates  $\theta, \phi$ .

# Linear Pattern

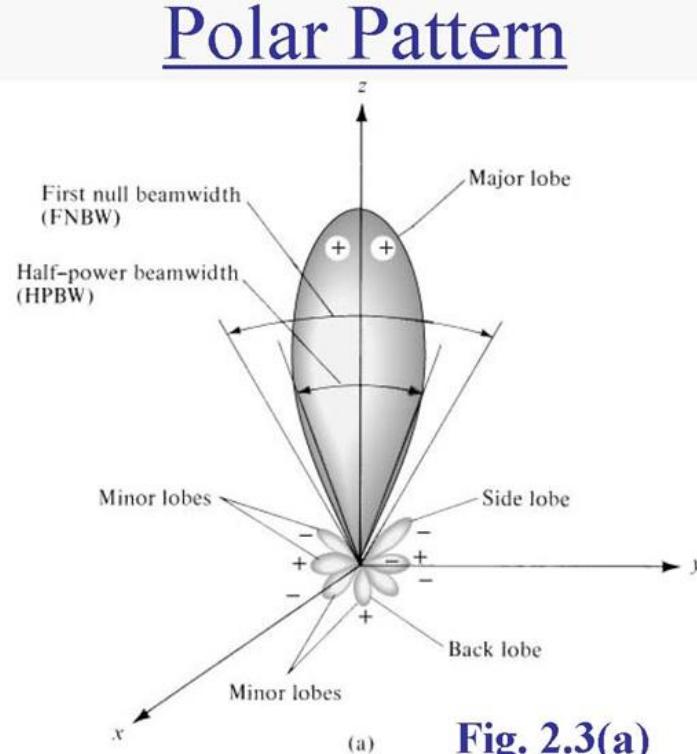
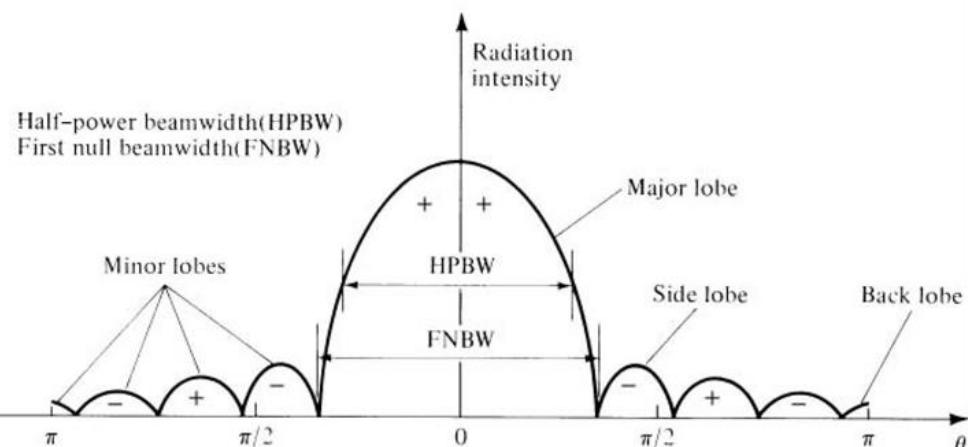
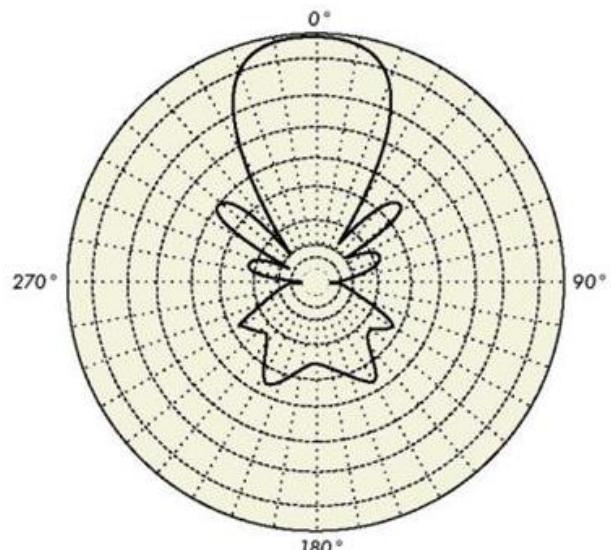


Fig. 2.3(a)

## Amplitude Radiation Pattern

- Field Pattern:  
A plot of the field (either electric  $|E|$  or magnetic  $|H|$ ) on a *linear* scale
- Power Pattern:  
A plot of the power (proportional to either the electric  $|E|^2$  or magnetic  $|H|^2$  fields) on a *linear* or *decibel (dB)* scale.



# Amplitude Radiation Pattern

Isotropic,

Directional,

Omnidirectional

## Directional Pattern of a Horn

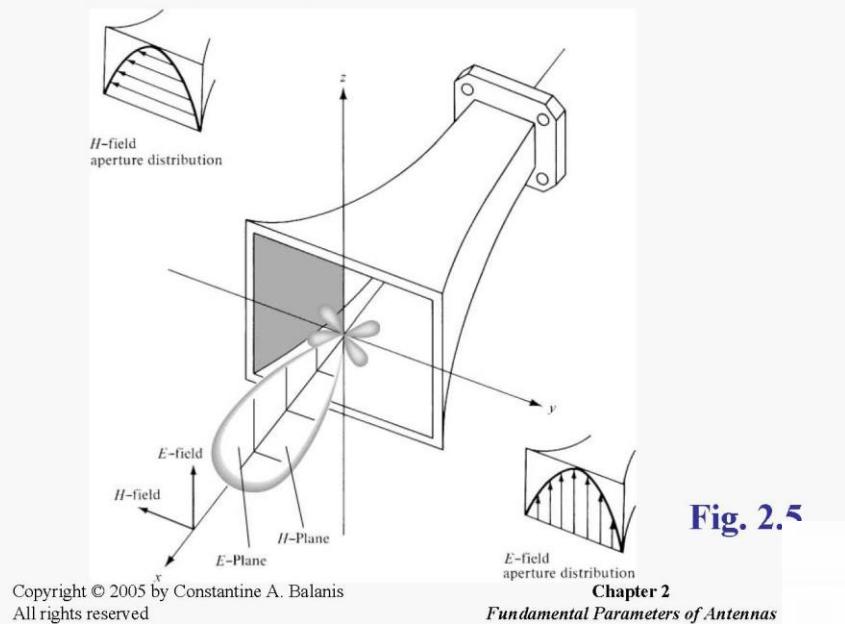
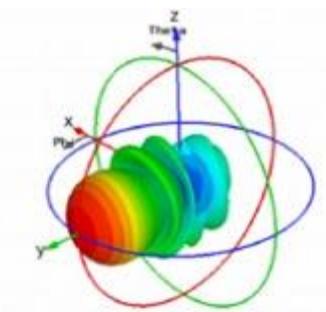


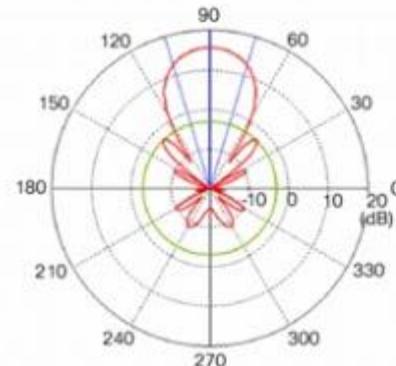
Fig. 2.5



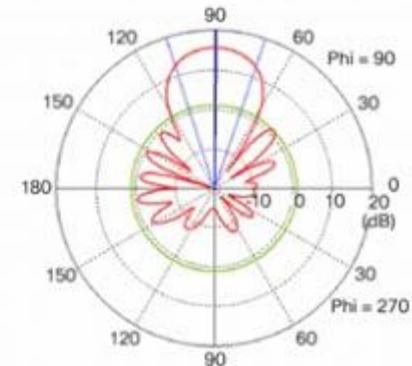
(a) Yagi Antenna Model



(b) Yagi Antenna 3D Radiation Pattern



(c) Yagi Antenna Azimuth Plane Pattern



(d) Yagi Antenna Elevation Plane Pattern

# Omnidirectional Pattern

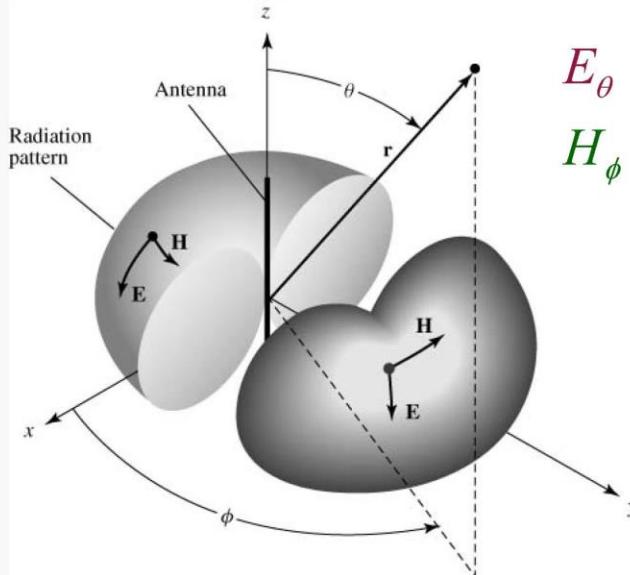
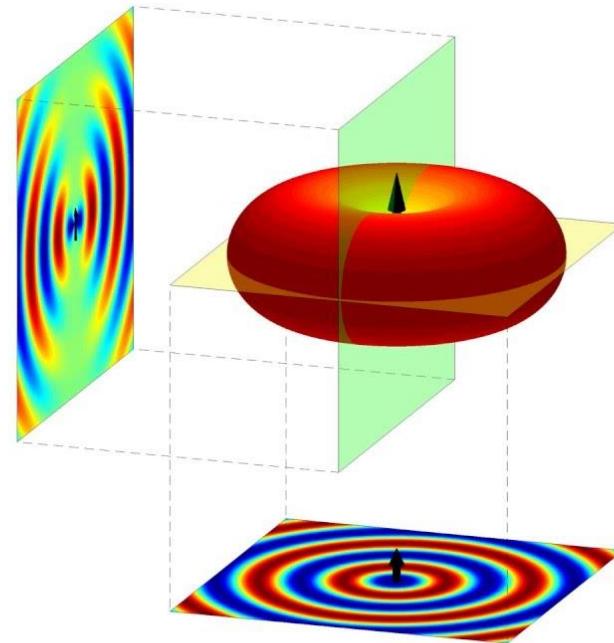


Fig. 2.6

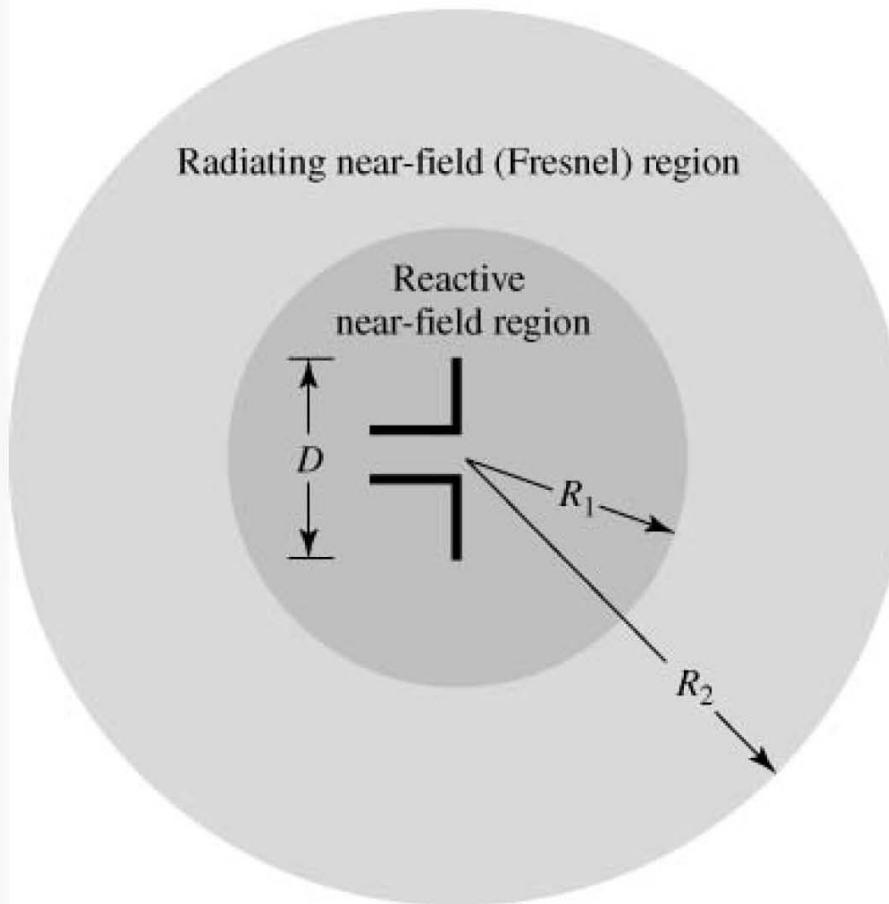
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# Field Regions

Far-field (Fraunhofer)  
region



$$R_1 = 0.62 \sqrt{D^3/\lambda}$$

$$R_2 = 2D^2/\lambda$$

**Fig. 2.7**

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# Field Regions

1. Reactive Near-field
2. Radiating Near-field: Fresnel
3. Far-field: Fraunhofer

## I. Reactive Near-Field

1. Highly reactive wave impedance
2. High content of non-propagating stored energy near the antenna

## II. Radiating Near-Field

Region where near-field measurements are made

## III. Far-Field

- A. Electric & magnetic fields are in-phase
- B. Wave impedance is, ideally, real
- C. Power predominantly real; propagating energy

# 1. Radiation Power Density

# 2. Radiation Intensity

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$$\underline{W}_{ave} = \frac{1}{2} \operatorname{Re} [\underline{E} \times \underline{H}^*] \quad (2-8)$$

It is assumed that both  $\underline{E}$  and  $\underline{H}$  represent peak amplitude values (not RMS: Root Mean Square). If  $\underline{E}$  and  $\underline{H}$  were to represent RMS values, then the one-half ( $1/2$ ) must be omitted. Measuring instruments typically measure RMS values.

$$P = \oint_S \underline{W}_{ave} \cdot d\underline{s} = \int_0^{2\pi} \int_0^\pi \underline{W}_{ave} \cdot \hat{a}_r r^2 \sin \theta d\theta d\phi$$

$$P_{rad} = P_{ave} = \int_0^{2\pi} \int_0^\pi \left( \frac{1}{2} \operatorname{Re} [\underline{E} \times \underline{H}^*] \right) \cdot \hat{a}_r r^2 \sin \theta d\theta d\phi \quad (2-9)$$

# Intensity $U$ (Far-field)

$$U = r^2 W_{rad} = r^2 W_{ave} \quad (2-12)$$

$$d\Omega = \frac{dA}{r^2} = \frac{r^2 \sin \theta \, d\theta \, d\phi}{r^2}$$

$$d\Omega = \sin \theta \, d\theta \, d\phi$$

$W$  = Power Density

$$= P / A \left( \frac{W}{m^2} \right)$$

$U$  = Radiation Intensity

$$= P / \Omega \left( \frac{W}{Sr} \right)$$

# Isotropic Source

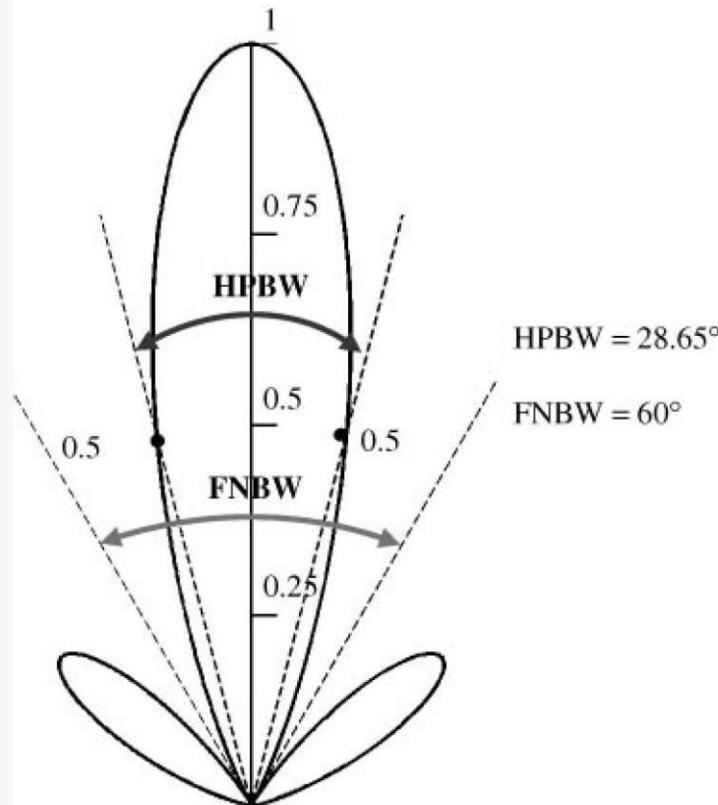
$$W_o = \frac{P}{4\pi r^2} \left( \frac{W}{m^2} \right)$$

$$U_o = \frac{P}{4\pi} \left( \frac{W}{Sr} \right)$$

# Beamwidths

- Half-Power (HPBW)
- First Null (FNBW)

# HPBW and FNBW of Radiation Intensity $U$



(b) Two-dimensional

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## Linear Scale

$$U(\theta, \phi) = \cos^2 \theta \cos^2 3\theta$$

Fig. 2.11(b)

# Directivity $D$

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$$D = \frac{U(\theta, \phi)}{U_o} = \frac{4\pi U(\theta, \phi)}{P_{rad}} \quad (2-16)$$

$$D_{\max} = D_o = \frac{U_{\max}}{U_o} = \frac{4\pi U_{\max}}{P_{rad}} \quad (2-16a)$$

$$D(dB) = 10 \log_{10} [D(\text{dimensionless})]$$

# Summary

1.  $D = \frac{4\pi U}{P_{rad}} = 1 = D_o$  (isotropic)
2.  $D = \frac{4}{\pi} \sin \theta = 1.27 \sin \theta = D_o \sin \theta$  (no specific one)
3.  $D = \frac{3}{2} \sin^2 \theta = 1.5 \sin^2 \theta = D_o \sin^2 \theta$  (infinitesimal dipole)
4.  $D = 1.67 \sin^3 \theta = D_o \sin^3 \theta$  ( $\lambda/2$  dipole)

# General Formulation of Directivity

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$$P_{rad} = \iint_S U(\theta, \phi) d\Omega = B_0 \int_0^{2\pi} \int_0^\pi F(\theta, \phi) \sin \theta d\theta d\phi \quad (2-20)$$

$$D_0 = 4\pi \frac{B_0 F_{\max}(\theta_m, \phi_m)}{\int_0^{2\pi} \int_0^\pi F(\theta, \phi) \sin \theta d\theta d\phi}$$

$$D_0 = \frac{4\pi F_{\max}(\theta_m, \phi_m)}{\int_0^{2\pi} \int_0^\pi F(\theta, \phi) \sin \theta d\theta d\phi} \quad (2-22)$$

$$D_0 = \frac{4\pi}{\int_0^{2\pi} \int_0^\pi \underbrace{\frac{F(\theta, \phi)}{F_{\max}(\theta_m, \phi_m)}}_{F_n(\theta, \phi)} \sin \theta d\theta d\phi}$$

$$D_0 = \frac{4\pi}{\int_0^{2\pi} \int_0^\pi F_n(\theta, \phi) \sin \theta d\theta d\phi} = \frac{4\pi}{\Omega_A} \quad (2-23)$$

$$\Omega_A = \int_0^{2\pi} \int_0^\pi F_n(\theta, \phi) \sin \theta d\theta d\phi = \text{Beam solid angle} \quad (2-24)$$

# Kraus

$$\Omega_A = \int_0^{2\pi} \int_0^\pi F_n(\theta, \phi) \sin \theta d\theta d\phi \simeq \Theta_{1r} \Theta_{2r}$$

$$D_0 = \frac{4\pi}{\Omega_A} \simeq \frac{4\pi}{\Theta_{1r} \Theta_{2r}} = \frac{41,253}{\Theta_{1d} \Theta_{2d}} \quad (2-26)$$

$$D_0 \simeq \frac{4\pi}{\Theta_{1r} \Theta_{2r}} = \frac{4\pi (180/\pi)^2}{\Theta_{1d} \Theta_{2d}} = \frac{41,253}{\Theta_{1d} \Theta_{2d}} \quad (2-27)$$

# Antenna Efficiency $e_o$

$$e_o = e_r [e_c e_d] = e_r [e_{cd}] \quad (2-44)$$

$$e_o = (1 - |\Gamma_{in}|^2) e_{cd} \quad (2-45)$$

$e_o$  = Total efficiency

$e_r$  = Reflection efficiency

$e_{cd}$  = Radiation efficiency

$e_{cd}$  = Radiation Efficiency

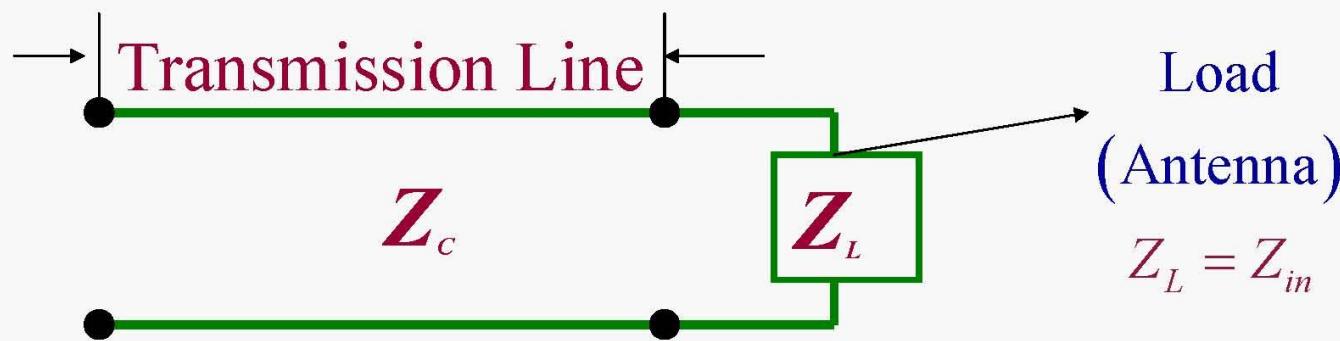
$$e_{cd} = \frac{\text{Power Radiated by Antenna } (P_r)}{\text{Power Delivered to Antenna } (P_r + P_L)}$$

$$e_{cd} = \frac{R_r}{R_r + R_L}$$

# Transmission Line and Load

$Z_c$  = Characteristic Impedance of Line

$Z_L$  = Load Impedance



$$\Gamma_{in} = \frac{Z_L - Z_c}{Z_L + Z_c} = \frac{Z_{in} - Z_c}{Z_{in} + Z_c}$$